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Evolution of transverse-momentum-dependent densities

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We discuss different operator definitions of the transverse-momentum dependent (TMD) parton densities from the point of view of their renormalization-group (RG) properties and UV evolution. We also consider the structure of the gauge links (Wilson lines) in these operator definitions and examine the role of the soft factor in the factorization formula within the TMD approach to semi-inclusive processes.

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The candidates for the *golden* measurement at the EIC are the spin-dependent Sivers function $f_{1T}^{\perp}$, as well as the unpolarized quark distribution f_1. The proposed *silver* candidates are the transversity, the Boer-Mulders, and the Collins functions. All these objects are transverse-momentum dependent (TMD) parton densities (termed in the following for simplicity, TMDs) that describe the inner structure of hadrons by taking into account the partonic longitudinal *and also* the transverse degrees of freedom [1–5].

The QCD factorization formula for a semi-inclusive structure function is expected to have the following symbolic form (modulo power corrections) [5–7]

$$F \sim H \otimes \mathcal{F}_D \otimes \mathcal{F}_F \otimes S \ . \tag{1}$$

This expression contains the hard part H, the distribution and fragmentation transverse-momentum dependent functions $\mathcal{F}_D$ and $\mathcal{F}_F$, respectively, and the soft part S, which is introduced in order to take care of light-cone singularities that cannot be cured by regularization. While the hard part H can be evaluated order by order within the framework of perturbative QCD, the TMDs are essentially nonperturbative entities and have to be modeled or extracted from the data. Thus, little can be said about these objects, except their general operator definition and their evolution behavior with respect to the energy and rapidity scales. Contrary to the fully collinear, i.e., *integrated* PDFs, the explicit operator definition of TMDs is quite problematic, owing to the appearance of extra (light-cone) singularities, and remains under active investigation (see, e.g., [8–17]).

Let us concentrate on the following two definitions[1] of an unpolarized quark TMD

$$\begin{aligned} f_1(x,\ \mathbf{k}_\perp|n) = \frac{1}{2}\int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} \mathrm{e}^{-ik^+\xi^- + i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp} \Big\langle h|\bar{\psi}_i(\xi^-,\boldsymbol{\xi}_\perp)[\xi^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\xi}_\perp]_n^\dagger \\ \times[\infty^-,\boldsymbol{\xi}_\perp;\infty^-,\boldsymbol{\infty}_\perp]_{\mathbf{l}}^\dagger\ \gamma^+\ [\infty^-,\boldsymbol{\infty}_\perp;\infty^-,\mathbf{0}_\perp]_{\mathbf{l}}[\infty^-,\mathbf{0}_\perp;0^-,\mathbf{0}_\perp]_n\psi_i(0^-,\mathbf{0}_\perp)|h\Big\rangle \end{aligned} \tag{2}$$

and its counter-part

$$f_1(x,\ \mathbf{k}_\perp|v) = f_1(x,\ \mathbf{k}_\perp|n\to v)\ . \tag{3}$$

following the so-called Amsterdam notations, e.g., [19], while other TMDs can be analyzed within the same framework. Gauge invariance is ensured by means of path-ordered contour-dependent Wilson-line operators (gauge links) with the

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[1] A generalized definition, which includes into the Wilson lines the spin-dependent Pauli term $F^{\mu\nu}[\gamma_\mu,\gamma_\nu]$, is worked out in Ref. [18]—see also [17].

generic form $[y, x|\Gamma] = \mathcal{P}\exp\left[-ig\int_{x[\Gamma]}^{y} dz_\mu \mathcal{A}^\mu(z)\right]$ where $\mathcal{A} \equiv t^a A^a$, and one has to distinguish between longitudinal $[\ ,\ ]_{[n,\, v]}$ and transverse $[\ ,\ ]_{[\mathbf{l}]}$ gauge links [9, 10]. On the other hand, the definition used in lattice simulations contains, in contrast to the (semi-)infinite Wilson lines in Eqs. (2) and (3), the finite, i.e., the *direct* gauge link joining the two quark fields [20–22].

In the tree-approximation, the "distribution of a quark in a quark" is normalized as

$$f_1^{\text{tree}}(x,\ \mathbf{k}_\perp|n) = \delta(1-x)\delta^{(2)}(\mathbf{k}_\perp)$$

and the integration over $\mathbf{k}_\perp$ yields formally the usual collinear (integrated) PDF

$$\int d^2k_\perp\ f_1(x,\ \mathbf{k}_\perp|n) = f_1^{\text{tree}}(x) = \delta(1-x)\ .$$

However, definition (2), taken literally, produces—beyond the tree-level—certain pathological divergences, which belong to one of the following three classes:

1. Usual *UV-singularities* $\sim \frac{1}{\varepsilon}$ from integrations over loop momenta, which can be removed by using the standard $R-$operation.

2. Pure *rapidity divergences*, which only appear in the *unintegrated* case. They cancel in the integrated distributions, but they are present in the TMD case giving rise to logarithmic and double-logarithmic terms of the form $\sim \ln\zeta,\ \ln^2\zeta$; they have to be resummed by a consistent procedure.

3. *Overlapping divergences*, which contain both UV and soft singularities simultaneously: $\sim \frac{1}{\varepsilon}\ln\zeta$, meaning that the UV pole ε^{-1} mixes with a "soft" divergence, regularized by the auxiliary parameter ζ.

In addition, one may encounter combinations of the above types of singularities ensuing from the soft factor. Apart from the UV divergences, all other singularities originate from uncompensated light-cone artifacts, which stem either from the lightlike gauge links (in covariant gauges), or from specific terms in the gluon propagator in (singular) light-cone axial gauges. While the singularities of the second class may be simply regularized by a rapidity cutoff—which is "separated" from other variables—the effect of the third class is more severe, because these singularities affect the UV-renormalization procedure, change the anomalous dimensions, and modify, therefore, the RG-evolution.

In order to avoid the above-mentioned problems, the following approaches have been proposed in the literature:

1. Shift in covariant gauges the gauge links off the light-cone: $v^2 \neq 0$, or use instead the non-lightlike axial gauge $(v \cdot A) = 0$ [5]. This amounts to definition (3), but it may cause problems in properly deriving factorization [11].

2. Stay on the light-cone, Eq. (2), but subtract some specific soft factor R, which is defined in such a way as to exactly cancel the extra divergences [23–25]: Eq. (2) is substituted by the "subtracted" function $f_1(n) \to f_1(n) \cdot R^{-1}$.

3. Perform a direct regularization of the light-cone singularities in the gluon propagator [26] $1/q^+ \to 1/[q^+](\eta)$, where η is an additional dimensional parameter [12]. In this case, a generalized renormalization is in order, which is formally equivalent to multiplying the TMD PDF by a particular soft factor [27].

4. Use the light-cone axial gauge, but supply it with the Mandelstam-Leibbrandt (ML) pole prescription [28, 29]: $1/q^+ \ \to\ 1/(q^+ + i0q^-)$ or equivalently $q^-/(q^+q^- + i0)$. Now the overlapping singularities do not appear from the outset—at least at the level of the one-loop order—while the contribution of the soft factor is reduced to unity, rendering the gauge-invariant definition valid [14].

Let us present the UV evolution equations for the above definitions. The off-the-light-cone TMD (3) does not contain *overlapping* singularities. Therefore, the renormalization-group equation reads [7, 30]

$$\mu\frac{d}{d\mu}\ f_1(x, \mathbf{k}_\perp, \mu|v) = \gamma_{\text{LC}}\ f_1(x, \mathbf{k}_\perp, \mu|v) \quad , \quad \gamma_{\text{LC}} = \frac{3}{4}\ \frac{\alpha_s C_{\text{F}}}{\pi} + O(\alpha_s^2)\ , \tag{4}$$

where γ_{LC} is the anomalous dimension of the bilocal quark operator in the light-cone gauge. If one factorizes out the soft contribution R_v, as proposed in Ref. [7], then the anomalous dimension changes and one has

$$f_1(v, \mu) \to f_1(v, \mu) \cdot R_v^{-1} \quad , \quad \mu\frac{d}{d\mu}\ \left[f_1(v, \mu) \cdot R_v^{-1}\right] = (\gamma_{\text{LC}} - \gamma_{\text{R}})\ f_1(v, \mu) \cdot R_v^{-1}\ , \tag{5}$$

where $\gamma_{\rm R}$ is the one-loop anomalous dimension of the soft factor R_v.

In contrast, the anomalous dimension of the "light-cone" TMD, before subtraction, deviates from $\gamma_{\rm LC}$ and this deviation is determined by the cusp anomalous dimension [12]:

$$\mu \frac{d}{d\mu}\ f_1(n) = (\gamma_{\rm LC} - \gamma_{\rm cusp})\ f_1(n)\ . \tag{6}$$

Hence, the generalized renormalization procedure restores the "broken" anomalous dimensions, so that one finds

$$f_1(n) \to f_1(n) \cdot R^{-1}\ \ ,\ \ \mu \frac{d}{d\mu}\ \left[f_1(n) \cdot R_n^{-1}\right] = \gamma_{\rm LC}\ \left[f_1(n) \cdot R_n^{-1}\right]\ . \tag{7}$$

In the light-cone gauge with the Mandelstam-Leibbrandt prescription, Eq. (2) supplemented by the soft factor R yields ab initio an anomalous dimension without lightlike artifacts:

$$\mu \frac{d}{d\mu}\ \left[f_1(n)^{\rm ML} \cdot R_n^{-1}\right] = \mu \frac{d}{d\mu}\ f_1(n)^{\rm ML} = \gamma_{\rm LC}\ \left[f_1(n)^{\rm ML} \cdot R_n^{-1}\right] = \gamma_{\rm LC}\ f_1(n)^{\rm ML}\ . \tag{8}$$

Thus, evolution equations can be established, while the evolution with respect to the *rapidity* variable—either ζ, or η (depending on the approach applied)—constitutes a separate task.

To conclude, let us sketch a couple of important but still un(re)solved problems.

(i) *Factorization* of semi-inclusive processes: an all-order factorization (in a covariant gauge) was studied in Ref. [7], but definition (3) was used which contains off-the-light-cone longitudinal gauge links. An explicit proof of a factorization theorem with lightlike longitudinal gauge links in the TMD PDFs is to our knowledge still lacking.

(ii) *Relationship between TMDs and collinear PDFs*: the generalized definition with lightlike longitudinal gauge links (7) does indeed yield after integration an $x-$dependent distribution function that obeys the DGLAP evolution equation

$$\int d^2 k_\perp\ f_1(x, \mathbf{k}_\perp, \mu | n) = f_1(x)\ ,\ \mu \frac{d}{d\mu}\ f_1(x, \mu) = \mathcal{K}_{\rm DGLAP} \otimes f_1(x, \mu)\ . \tag{9}$$

The reason is that the overlapping singularities in the real and the virtual gluon contributions cancel against each other after carrying out the $\mathbf{k}_\perp$-integration. In contrast, for an axial but not-lightlike gauge, one gets a function containing longitudinal gauge links off-the-light-cone along the vector $v = (v^+, v^-, \mathbf{0}_\perp)$, i.e.,

$$\int d^2 k_\perp\ f_1(x, \mathbf{k}_\perp, \mu | v) = f_1(x, \mu | v)\ \ ,\ \ \mu \frac{d}{d\mu}\ f_1(x, \mu | v) = \mathcal{K}_v \otimes f_1(x, \mu | v)\ \ ,\ \ \mathcal{K}_v \neq \mathcal{K}_{\rm DGLAP}\ . \tag{10}$$

The RG-properties of this object differ from those of the TMD PDF with lightlike longitudinal gauge links (which fulfills the DGLAP equation shown above). This difference leads, in particular, to different evolution equations and different phenomenological implications.

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